Non-Scalar Economy: An Incentive-Based Approach for Managing Pollution and Attaining Economic Sustainability

Farzad Safaei, University of Wollongong, Australia, farzad@uow.edu.au

Abstract

In the current economy, the cost of damage to the environment is often *external*, which is likely to lead to over-exploitation and inadequate provisioning. There are a variety of incentive-based schemes, such as effluent taxes or tradable permits, which attempt to confront the polluting agents with a 'price' equal to the marginal external cost of their activity. However, competitive incentives alone are not adequate for effective management of the shared resources. Thus, the current solutions exhibit several drawbacks including susceptibility to international *free-riding*, sensitivity to accurate estimation of *cost* of pollution or environmental *targets*, and inability to take the *cumulative* nature of pollution and its cost into account.

This paper proposes an approach to *eliminate* the commons dilemma by using non-scalar numbers for the underlying economic signals. Rather than *adding* the cost of pollution to the cost of private resources, this cost is kept in a separate dimension represented by the second component of money and price. This separation enables us to develop more effective models for economic signals and incentives, and avoid the above-mentioned drawbacks. In the proposed design, the cost of pollution is cumulative and would have a *progressively higher* economic impact on *both* the competitive and cooperative incentives for managing pollution leading to a *provable* sustainable point. Moreover, the proposed model does not suffer from the *free rider* problem; does not require accurate estimation of the *true cost* of pollution; is *simple* to implement and *backward compatible* with the current economy.

Key Words: Environmental Economy, Global Commons, Non-Scalar Economy, Incentive-Based Schemes, Sustainability.

1 Introduction

The global environmental resources, such as the atmosphere and the oceans, have been used as 'waste-sinks' for many years and the level of pollution is commonly viewed to be excessive. The current economy is reasonably effective in managing scarce resources that are privately owned. However, for global commons, the cost of damage is *external* and the appropriators continue to engage in pollution until its marginal benefit is zero [1]. This is likely to lead to over-exploitation, inadequate provisioning and perhaps irreversible damage. In essence, there is a failure of the doctrine of 'invisible hand' and economic agents face a 'commons dilemma' since pursuit of their self-interest may contravene the common good [2].

A variety of incentive-based schemes, such as effluent taxes or tradable permits, have been proposed to 'internalize' this cost and confront the polluting agents with a 'price' equal to the marginal external cost of their activity [1][3]. However, with respect to management of global resources, there still remain some key weaknesses as outlined below:

a) **International free-riding:** The commons dilemma favors free-riding as the dominant strategy, which makes *partial* agreements ineffective. It is often claimed that compliance reduces the international competitiveness of one's country if some other countries do not comply [4]. The polluting firms could also relocate their operation to such countries creating significant political pressure. Consequently, implementation of the above schemes requires global consensus and enforcement for effectiveness.

b) **Sensitivity to accurate evaluation of cost of pollution:** For incentive-based schemes to result in pollution reduction, the marginal cost of pollution should exceed the marginal abatement cost. However, it is politically difficult to impose high costs on polluters or assess the true cost of pollution because various competing parties are strongly motivated to give distorted estimates [6][7]. International agreements on environmental targets also involve intense lobbying and prolonged negotiations. As a result, the policy objectives are often set conservatively with suboptimal outcome [3][9][10].

c) **Cumulative nature of pollution:** The impact of pollution in the real world is cumulative across time, space and multiple uses, which is not reflected in current schemes[5][11]. A particular polluting activity *may indeed have negligible impact and cost* in isolation, but when it is occurring after centuries of pollution, as part of a large community of polluters and compounded by other activities that also lead to pollution of the same resource, its impact may become significant. In other words, *history matters* and the system cannot be memory-less. The cost of pollution imposed on an economic agent must be an *increasing* function of how much pollution is created by others or this same agent in other contexts.

The objective of this paper is to *eliminate the commons dilemma* by introducing a new concept of money and price based on two-dimensional numbers. One of these dimensions is responsible for managing private property and the other deals with the globally shared environmental resources. Rather than *adding* the cost of pollution to the cost of private resources, as is common in current incentive-based schemes, this cost is kept in a separate dimension represented by the second component of money and price. This separation enables us to develop more effective models for economic signals and incentives, and avoid the above-mentioned pitfalls and limitations.

For example, in the proposed model the marginal cost of pollution can be controlled without the need to accurately estimate the cost of pollution. Moreover, this marginal cost can vary among the individuals or communities depending on their past polluting behavior and the level of accumulation of pollution in the economy. Another feature of the proposed design is that during a purchase operation, the pollution liability represented by the second component of price is *transferred* to the buyer's money from the seller. The buyers, therefore, bear the economic consequences of supporting the pollution. However, supporting a polluting product *does not reduce one's wealth* or cause *transfer* of wealth to another party. Since no one benefits from pollution, there would be no perverse incentives within the economy and the cooperative incentives for pollution abatement are strengthened.

This conceptual change to non-scalar money and price has a profound impact on *both* competitive and cooperative incentives for managing pollution. In particular, the proposed model leads to a *provable* equilibrium point, in which the level of per-capita pollution accumulated in the economy is kept at a constant level. Moreover, it is easy for the society to change this level by adjusting a single macro-economic parameter, thereby steering the economy towards a sustainable point (Section 5). In addition, we will demonstrate the following properties for the proposed design:

- **No free-riding:** Countries that do not participate in the scheme would be economically disadvantaged - Section 6.2.
- **The cumulative nature of pollution is taken into account:** The cost of pollution is cumulative and would have *progressively higher* economic impact - Section 5.
- **No dilemma:** There is no conflict between individual desire for economic success and discharging one's obligations towards the environment - Section 7.
- **No need for accurate estimation of cost of pollution:** Sections 3.2.1 and 5.

- **Practical to implement and backward compatible:** There is no need for a disruptive introduction as the proposed economy is backward compatible with the current system and a smooth migration is possible - Section 6.1.
- **Simplicity of basic operations:** The basic mathematical operations of the economy (Section 2) are simple and can be mastered during primary school years.

To limit the scope of this article, the formal mathematical proofs are not included in the current text. These will be subject of future publications and can be obtained directly from the author.

2 Mathematical Definitions, Operations, and Properties

In this Section, a new class of two-dimensional numbers is introduced to be used as the basis of the proposed non-scalar money and price. A member of this class, such as $\mathbf{x} = (x_1, x_2)$, has two components. We require that *both components be real numbers and the first component be non-negative*. The set of these numbers could be viewed as coordinates of points on the two-dimensional *half*-plane of real numbers (the other half is excluded because the first component cannot be negative), see Figure 1.

The first component, x_1, is called the *Wealth Component* (WLC) and is responsible for managing scarce resources that are privately owned. Accordingly, it performs a role similar to the scalar money and price and current intuitions about its meaning and function are largely applicable.

The second component, x_2, is called the *Pollution Liability Component* (PLC), which manages the scarcity of globally shared resources of interest. Note that current understanding about the meaning of 'liability' *could be misleading* and the PLC role becomes clear as the properties of non-scalar economy are investigated further in this Section.

Both of these components have the same units of currency, such as dollars. We refer to an economy based on non-scalar money and price as *non-scalar economy*, in contrast to the current (scalar) economy.

Example – price: In non-scalar economy, the price of an item may be $(10, 1.60). This means that the seller is asking for $10 of Wealth Component (WLC) to compensate for all the privately owned resources, such as labor, materials, plant, used throughout the supply chain for this product up to this phase. It also means that the seller is claiming that $1.60 of Pollution Liability Component (PLC) was incurred in *total* by all the entities involved (extraction of raw materials, electricity generation, transport, refrigeration, etc.) to get the product to current stage. It is shown in Section 4 that sellers would have very strong economic incentives to represent the PLC of price as accurately as possible otherwise they bear an economic cost. It is also shown that this accurate estimation is very *easy* for them. As a result, buyers can be confident that the PLC of price is reliable.

Example – money: In non-scalar economy, the balance of someone's saving account might be $(100,5). This means that the owner has accumulated $100 of Wealth Component and $5 of Pollution Liability Component in this account. The role and impact of the latter on the owner's buying power will be discussed later in this section.

2.1 *The impact of PLC: Index of money and price*

Our aim is to find a suitable metric to measure how *prominent* the second component (PLC) of money and price is with respect to the first component. As mentioned before, a mere comparison of magnitudes is not a good indicator. Hence, in this paper for every $\mathbf{x} = (x_1, x_2)$, be it price or money, we use the ratio $x_2/(x_1 + x_2)$ for this purpose. Let us call this metric the *Liability Index* of $\mathbf{x}$ (or *Index* of $\mathbf{x}$ for short). When the PLC is zero or negative, we *define* the Index to be zero because a negative Index is not a useful concept. Therefore, the *Liability Index* of $\mathbf{x}$, denoted by $I(\mathbf{x})$, is defined as follows:

$$I(\mathbf{x}) \equiv \begin{cases} \dfrac{x_2}{x_1 + x_2} & \text{if } x_2 > 0 \\ 0 & \text{if } x_2 \le 0 \end{cases}$$

Based on the above definition, the Index of any money or price is always between 0 and 1. The closer the Index to one, the more prominent is the PLC.

2.2 The Limit on the Index of money

It is not appropriate to impose any limits on the Index of a price, because we would like the price to reflect the 'reality' – that is, the true combination of cost of private and shared resources used in production. However, *for money,* the economy should impose a limit on how large the Index could be. This limit - called the *Liability Index Threshold* - is mathematically enforced on all the

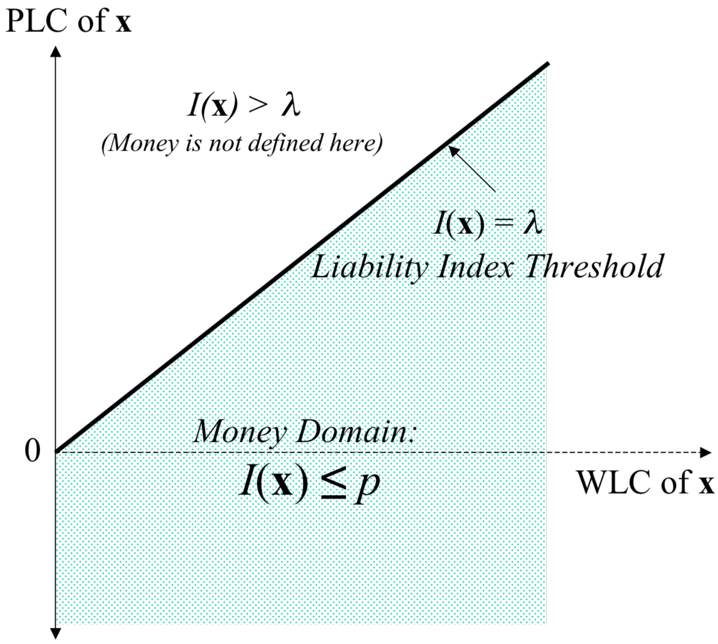

Figure 1: Two-dimensional half-plane for money and price, Money Domain and LI Threshold

operations of the economy ensuring that *the Index of money is always below this threshold* (see Section 2.3). The Liability Index Threshold, or LI Threshold for short, is a number between 0 and 1, excluding zero, which is assigned to every economic agent, such as an individual, a firm, or a government department, by the regulator. Also, we will see that the regulator is bound to confine the Liability Index Thresholds within a certain range based on the environmental efficiency of the country's economy (see Section 3.3).

In Figure 1, the locus of all non-scalar numbers with Index equal to the Liability Index Threshold λ is shown as a thick solid line. For someone whose LI Threshold is λ, the Money Domain is shown as a shaded area below and including this line. The Index of any number above this area would be above the LI Threshold and hence these numbers cannot be money.

2.3 Defined operations for money and price

The following mathematical operations are defined for the non-scalar economy.

1- Addition: The familiar component-wise addition is defined for money and price as follows:

$$(x_1, x_2) + (y_1, y_2) \equiv (x_1 + y_1, x_2 + y_2)$$

This operation could be used to add prices, for example, prices of items in a shopping trolley to get the total price; or add monies, for example, different account balances to get one's total money. It is *not* meaningful to add a price to money or vice versa.

2- Subtraction: The familiar component-wise subtraction is defined for non-scalar money and price, *provided the result is valid*:

$$(x_1, x_2) - (y_1, y_2) \equiv (x_1 - y_1, x_2 - y_2)$$

When subtracting prices from each other, the result must be a valid price, that is, with non-negative WLC. In case of money, for example, withdrawing money from an account, the result must be *valid money*, that is, *its Liability Index should not exceed the LI Threshold*.

It is *not* meaningful to subtract a price from money or vice versa.

3- Purchase: In the current economy, the purchase operation is a subtraction of price from tendered money. In non-scalar economy, the purchase operation is *not* subtraction, but still very simple. Let the buyer's money before the purchase be **x** = (x_1, x_2) and assume that buyer intends to purchase a product priced at **a** = (a_1, a_2). The buyer's money after the purchase is obtained by performing a *purchase operation* on **x** and **a** as defined below.

$$(x_1, x_2) \, ⓟ \, (a_1, a_2) \equiv (x_1 - a_1, x_2 + a_2)$$

The purchase operation can take place *provided the result is valid money*. In other words, the Index of (x_1 - a_1, x_2 + a_2) must not exceed the *buyer's* LI Threshold. Otherwise, we say that **x** does not have sufficient funds to make this purchase. In the example of Figure 2, $**x** can purchase an item priced at $**a**, but does not have sufficient buying power to purchase $**b**.

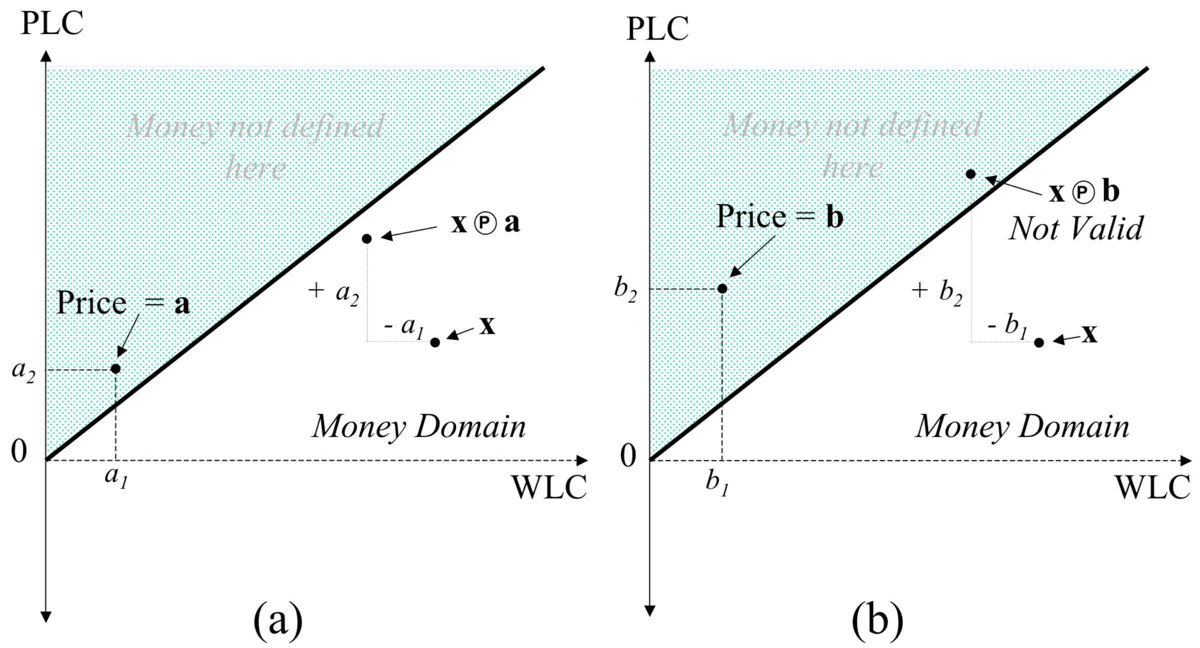

Figure 2: (a) **x** can purchase **a**; (b) **x** cannot purchase **b**

4- Sell: Assume that seller's money before the sale is **w** = (w_1, w_2). After selling an item priced at **a** = (a_1, a_2), the seller's money can be calculated as follows:

$$(w_1, w_2) \, ⓢ \, (a_1, a_2) \equiv (w_1 + a_1, w_2 - a_2)$$

Again, the result of a sell operation must be valid money.

The interpretation of purchase and sell operations is straightforward. The buyer *pays* for the Wealth Component of price to compensate the seller for all the privately owned resources used in producing the good. The buyer also *accepts* the Pollution Liability Component of price, which is transferred to his money and deducted from seller's.

The above set of definitions and operations *completes* the mathematical design of the non-scalar economy. The rest of this paper attempts to explore the ramifications of living in an economy based on this design and discusses practical implementation issues.

2.4 Buying Power of money and Magnitude of price

As shown in Figure 2, a buyer is able to purchase a good provided the result of the purchase operation is valid money. To check affordability, therefore, one can use this operation and examine the result to see if the purchase can take place. But money and price have two components and the interplay between these components in determining how expensive the price is, or the amount of buying power in the money may not be obvious. To assist with understanding and also analysis, in this section we introduce two useful quantities for this purpose.

Buying Power of money: Consider money $\mathbf{x} = (x_1, x_2)$ shown in Figure 3-(a). After any purchase, the buyer's money gets 'closer' to the Liability Index Threshold[1] (see Figure 2). In fact, if the buyer's money is already *on* the LI Threshold – that is, its Index is equal to the LI Threshold - *it cannot buy anything*. For example, money $\mathbf{y}$ in Figure 3-(a) still contains wealth (in the sense that its Wealth Component, WLC, is not zero), but does not have any buying power. Consequently, the 'distance' of money from the LI Threshold is a good measure of the amount of buying power. Formally, we define the Buying Power of money as the Wealth Component of the maximum price of the form $(B,0)$ that can be purchased. Geometrically, this is the *length of the horizontal line segment between the money and the LI Threshold* as shown in Figure 3-(a). Mathematically, the *Buying Power* of money $\mathbf{x}$ with respect to LI Threshold λ, denoted by $B_\lambda(\mathbf{x})$ is obtained as follows:

$$B_\lambda(\mathbf{x}) = \begin{cases} x_1 - \rho\, x_2 & \text{if } x_2 > 0 \\ x_1 & \text{if } x_2 \le 0 \end{cases}; \quad \text{where} \quad \rho \equiv \frac{1-\lambda}{\lambda}.$$

Note that Buying Power of money is always evaluated with respect to its owner's LI Threshold. Also note that Buying Power *can never exceed the Wealth Component of money*. In other words, there is *no Buying Power in the PLC component of money* and even if the PLC is negative or zero, the Buying Power at most becomes equal to WLC. For example, if $\lambda = 0.4$ ($\rho = 1.5$), the Buying Power of \$(100,5) is \$92.5.

Magnitude of price: Similarly, we can define a scalar quantity, referred to as *Magnitude of price*, to measure how 'expensive' a price is for the buyer. The Magnitude of price $\mathbf{a} = (a_1, a_2)$ with respect to the buyer's Liability Index Threshold λ, denoted by $M_\lambda(\mathbf{a})$, is defined as the *amount of Buying Power lost in purchasing this price* when the buyer's money has non-negative PLC. It can be shown that the Magnitude is obtained by the following equation:

$$M_\lambda(\mathbf{a}) = a_1 + \rho\, a_2$$

It can be proved that a buyer is able to purchase an item if the Buying Power of his money is greater than, or at least equal to, the Magnitude of price – both evaluated with respect to the buyer's Liability Index Threshold. This is because of the following theorem:

Theorem 1: Purchase operation on money $\mathbf{x}$ and price $\mathbf{a}$, both with non-negative PLC, is valid if and only if $B_\lambda(\mathbf{x}) \ge M_\lambda(\mathbf{a})$.[2] (Proof is straightforward and omitted for brevity.)

[1] This statement applies to buying *ordinary* goods and not the *environmental goods* described in Section 4. Purchase of environmental goods *increases* one's Buying Power and moves money 'away' from the LI Threshold.

[2] If the PLC of money is negative, $\mathbf{x}$ can absorb certain level of PLC in the price without losing Buying Power, but as soon as PLC of money becomes zero the above theorem holds.

For example, if $\mathbf{x}$ = \$(50, 10) and $\mathbf{a}$ = \$(35, 8), for $\lambda = 0.6$ Buying Power of $\mathbf{x}$ is \$43.3 and Magnitude of $\mathbf{a}$ is \$40.3 (so $\mathbf{x}$ can purchase $\mathbf{a}$). For $\lambda = 0.5$, Buying Power of $\mathbf{x}$ is \$40 and Magnitude of $\mathbf{a}$ is \$43 (so $\mathbf{x}$ cannot purchase $\mathbf{a}$).

The above theorem specifies a *necessary and sufficient* condition; hence the definitions of Buying Power and Magnitude provide a rigorous basis for determining affordability.

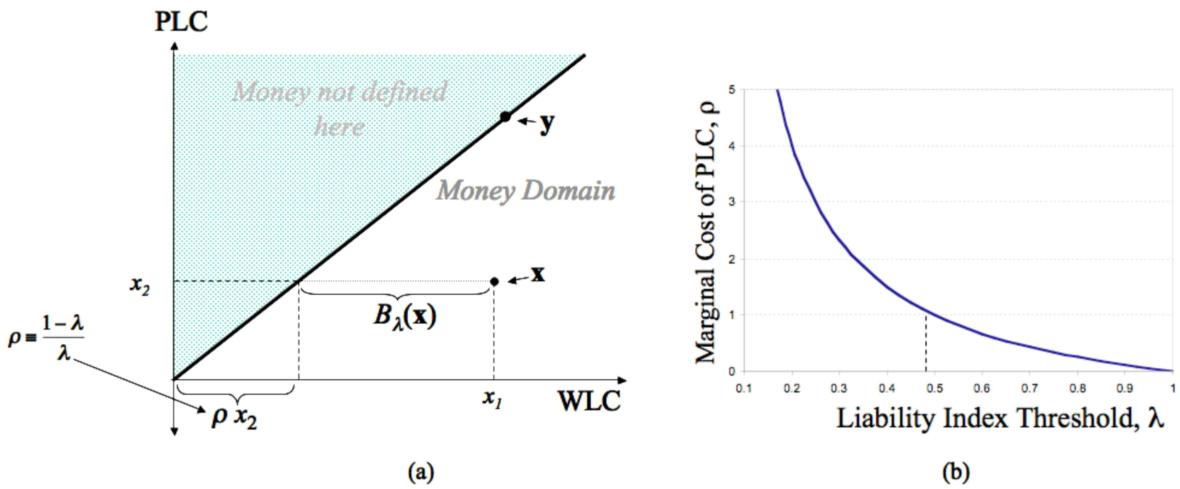

Figure 3: (a) Buying Power of money; (b) relationship between Marginal Cost of PLC and LI Threshold

2.5 *Marginal Cost of PLC*

The parameter ρ in the expressions for Buying Power of money and Magnitude of price plays a critical role in the proposed non-scalar economy. Hereafter, this parameter is referred to as the *Marginal Cost of PLC* [3] and represents the impact of PLC on Buying Power of money and Magnitude of price.

To illustrate this point, consider money $\mathbf{x}$ with positive PLC. The Buying Power of $\mathbf{x}$ is equal to $x_1 - \rho\, x_2$. This means that for every extra dollar of PLC, the Buying Power is reduced further by ρ dollars[4]. Note that the total wealth contained in money is *intact*. In other words, this is not a 'pollution tax' that the owner has paid. The presence of PLC only makes a certain amount of Buying Power inaccessible. Hereafter, we say that some of the Buying Power is *locked up* against the PLC. This locked up Buying Power could be released (made accessible to the owner) if the PLC is reduced. By reducing every dollar of PLC, ρ dollars of Buying Power is released, up to the point when PLC becomes zero. So ρ is also the *marginal benefit of reducing* PLC. Likewise, the Magnitude of price, $a_1 + \rho\, a_2$, increases by ρ dollars for every additional dollar of PLC in the price[5].

Hence, the Marginal Cost of PLC, ρ, measures the impact of PLC in reducing Buying Power on one hand and increasing the perceived Magnitude of price on the other hand. The Marginal Cost of PLC depends on the LI Threshold and, therefore, in general it would vary among the population. The

---

[3] Our terminology is rather loose here for convenience. To be precise, parameter ρ should be called the *Lower Bound* on Marginal Cost of PLC because the prospect of changes in the Liability Index Threshold increases the Marginal Cost of PLC above this value (Section 5). But for a short period of time, when the Liability Index Threshold is fixed, ρ is applicable. In general, there is no relationship between Marginal Cost of PLC and the actual marginal cost of pollution. However, at the sustainable point it may be argued that these two coincide.

[4] Mathematically for Buying Power: $\dfrac{\partial B_\lambda(\mathbf{x})}{\partial x_2} = -\rho$, when $x_2 > 0$ and λ is constant.

[5] Mathematically $\dfrac{\partial M_\lambda(\mathbf{a})}{\partial a_2} = \rho$.

relationship between ρ and λ is shown in Figure 3-(b). This non-linear relation is very useful as it allows ρ to assume any positive value and enables us to control ρ by altering the LI Threshold individually (see Section 3.3).

2.6 Summary and comparison

To summarize the discussions of this Section, consider a simple example regarding the annual income and expenses of a family (Table 1). The gross annual salary of this family is \$(85000, 0). The PLC of salary is assumed to be zero because, typically, the pollution liability associated with the operation of employer's business is transferred to the customers (via the sell operation) or shareholders (via dividends) and not to employees – see Section 4.

The annual tax imposed on this family is \$(35000, 800). The government would accumulate PLC while running its services and operations. Thus, the Liability Index of tax is calculated and applied to all collected taxes. The tax 'deduction' in non-scalar economy is a *purchase operation*. In essence, we are purchasing our share of government services. The net annual income after tax is:

$$\$(85000, 0) ⓟ \$(35000, 800) = \$(50000, 800).$$

Table 1 shows the annual prices for various expenses. Each firm estimates the PLC of its price to avoid accumulation of PLC within the firm (Section 4). If the family were able to buy these goods, their net balance would be \$(3800, 2200). In the Table, three cases are shown with different Liability Index Thresholds. For each case, the Magnitude of prices and the final Buying Power left for the family are shown. For case 3, the LI Threshold is such that the family cannot afford to buy all of these products. So either they should reduce their consumption or shop around to substitute by more affordable products.

For the first two cases, this family accumulates \$2200 of PLC during this year, which locks up \$2200 or \$3300 of their Buying Power for LI Thresholds of 0.5 and 0.4 respectively. If they do nothing to reduce this PLC, after several years of accumulation, potentially a significant amount of Buying Power could become inaccessible. However, *their wealth still belongs to them* and is in their bank account so their incentive to release this locked up Buying Power grows stronger. It is shown later (Section 3.3) that their LI Threshold may also be lowered. With a lower LI Threshold, the Marginal Cost of PLC goes up and more Buying Power is locked up against the same level of PLC. The incentive to reduce PLC grows stronger still.

The Table also contains a comparison with an 'equivalent' situation in the scalar economy, whereby the same dollar values for cost of pollution (taxes or cost of permits) have been added to the price. In this case, the family pays \$2200 of 'tax' regardless of their previous history. The cost of pollution is buried in the prices and is not visible to the buyer. Payment of these pollution charges generates revenue for others and the family would hope that the agency who received the revenue will do something to undo the damage to the environment, but *this is not enforced by the economy*. There is no visible accumulation of the environmental cost and no economic impact specific to accumulation, *so if the cost is not significant at this stage, it would be swamped by other concerns and is unlikely to cause a change of behavior*.

	Non-Scalar Economy					Scalar Economy	
Description	**Value (\$)**	**Operation**	**Case 1:** $\lambda = 0.5$, $\rho = 1.0$	**Case 2:** $\lambda = 0.4$, $\rho = 1.5$	**Case 3:** $\lambda = 0.3$, $\rho = 2.3$	**Value (\$)**	**Operation**
Gross Salary	(85000, 0)		Buying power = \$85,000	Buying power = \$85,000	Buying power = \$85,000	\$85,000	
Tax	(35000, 800)	Purchase	Magnitude = \$35,800	Magnitude = \$36,200	Magnitude = \$36,840	\$35,800	Subtraction
Electricity	(1200, 300)	Purchase	Magnitude = \$1,500	Magnitude = \$1,650	Magnitude = \$1,890	\$1,500	Subtraction
Car and Petrol	(3000, 200)	Purchase	Magnitude = \$3,200	Magnitude = \$3,300	Magnitude = \$3,460	\$3,200	Subtraction
Other expenses	(42000, 900)	Purchase	Magnitude = \$42900	Magnitude = \$43, 350	Magnitude = \$44, 070	\$42,900	Subtraction
Balance	**(3800, 2200)**		**Buying power = \$1,600**	**Buying power = \$500**	**Not Possible**	**\$1,600**	

Table 1: Comparison of income and expenses for a family in both economies

3 Creation and Removal of Pollution Liability

In non-scalar economy, operations of purchase and sell result in *transfer* of PLC from one economic entity to another. Thus, the pollution liability *circulates* in the economy. However, it has to be *created* because of appropriation, and *removed* from circulation due to provisioning activity.

It is proposed here to have a separate entity charged with overseeing the creation and removal of pollution liability. For management of privately owned resources, the economy needs external organizations to enforce ownership rights. Because the global commons are shared, there is no need for a legal and judiciary system to enforce ownership. What is required is an independent *accounting* authority to keep track of appropriation and provisioning activities. Let us refer to this as the *Accounting Authority for Pollution Liability* (AAPL).

AAPL is created by all those countries that have decided to adopt the non-scalar economy. AAPL is independent of the government and is not a 'regulator'. Its role is not to prescribe certain practices and enforce environmental standards or targets. It merely keeps track (account) of appropriation and provisioning levels.

AAPL is responsible for three primary functions: (i) creation of pollution liability as a result of appropriation and assigning this liability to the appropriator; (ii) removal of pollution liability as a result of provisioning activity; and (iii) imposing a suitable Liability Index Range (LI Range) on each participating country based on an internationally agreed formula. These functions are briefly described below.

3.1 Creation of Pollution Liability

AAPL monitors the appropriation of global resources and, in response, assigns suitable values of PLC to the appropriator. The *price* of appropriation is of the form (0, b), which the *appropriator has to purchase*. Note that the Wealth Component of this price is zero. Hence, AAPL does not receive any revenue (Buying Power) as a result of pollution. AAPL does not impose any ceiling on the amount of appropriation of a resource. It calculates and assigns PLC based on every unit of resource, e.g. one cubic meter of air or water polluted.

3.2 Removal of Pollution Liability

Let us define any activity that attempts to undo the effect of pollution and restore the resource to its original condition as *provisioning* function[6]. In the non-scalar economy, the entity engaged in provisioning can transfer some PLC to AAPL. This transfer can be viewed as a sell operation which has a price in the form of (0, c). The PLC transferred to AAPL after this operation is *removed from circulation*. Once again, there is no transfer of Wealth Component during removal to/from AAPL. Different scenarios for PLC removal can be envisaged. For example:

a) **Individual or community initiatives:** These initiatives may involve restoration, clean up, waste management and similar activities accredited by AAPL.

b) **Environmental Firms:** The core competency of these firms is to restore the damage caused by pollution. The 'product' sold by these firms is called an *environmental good* and has a price with *negative* PLC of the form (e_1, -e_2)[7]. By purchasing an environmental good, the buyer loses \$$e_1$ of WLC, with a corresponding Buying Power loss of \$$e_1$, and also reduces his/her PLC by \$$e_2$, which may increase Buying Power by up to \$ $\rho\, e_2$. Depending on buyer's LI Threshold this purchase could have a positive return and increase the buyer's Buying Power. In addition, the environmental firms may return dividends with *negative* PLC to their shareholders.

[6] We distinguish between *abatement* and *provisioning* activities. Pollution abatement reduces the rate of creation of PLC during manufacturing or delivery of a service. Provisioning activity is concerned with clean up after the pollution has been created. The environmental sector in non-scalar economy could have both of these activities.

[7] We use the negative sign to emphasize that the PLC of price is negative. That is, e_2 is the absolute value of PLC.

c) **Discounting due to natural regeneration:** An agreed level of PLC reduction is *applied to all* on regular time intervals to represent the natural regeneration capacity of the environment.

Figure 4 shows a representation of PLC creation and removal functions as a 'leaky bucket'. The PLC is created as a result of appropriation and flows into the economy. The rate of in-flow of PLC can be reduced by abatement activities and better technology. The PLC circulates in the economy and as the in-flow continues the level of accumulation (the depth of the bucket) increases. At the same time, the bucket is leaking - there is some outflow due to natural discounting and some due to provisioning, if any. The environmental sector influences the amount of PLC in circulation via abatement and provisioning activities. It is shown in Section 5 that the proposed non-scalar economy *always* reaches an *Equilibrium Point*, at which the bucket depth remains at a constant level. It is also shown that it is very easy for the society to control the depth of bucket at the Equilibrium Point by modifying a single macro-economic parameter.

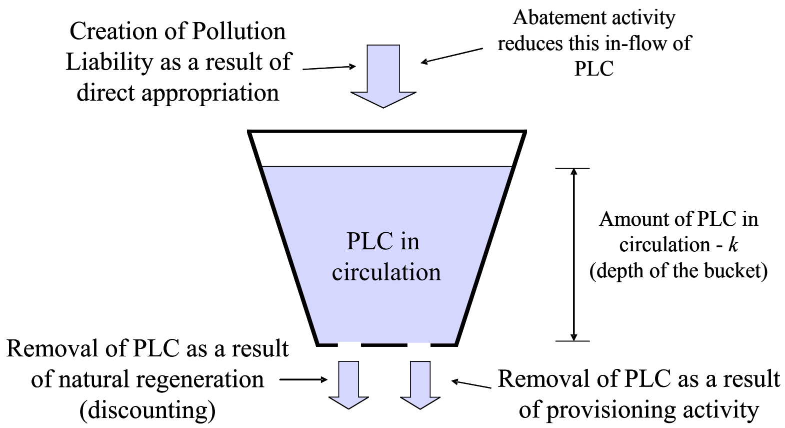

Figure 4: Creation and removal of PLC and the impact of environmental sector

3.2.1 The unit price of environmental resources

During creation and removal of PLC, AAPL has to put a dollar value on a 'unit' of polluted resource. For example, if atmosphere and water are selected to be managed by the non-scalar economy, there is a *unit price* associated with appropriation of one cubic meter of air or water - say, \$(0, *a*) and \$(0, *w*) respectively.

One way to incorporate different pollutants into this model is to agree on a 'safe' level of concentration of each pollutant per unit of resource. If the appropriator emits this safe level, one unit of resource has been made unavailable to others to use as waste-sink.

A key feature of the non-scalar economy is that it is *not* sensitive to the magnitude of unit prices, such as *a* and *w* in the above example. Any number will do as long as it is applied consistently. The reason is that the *magnitude of PLC by itself does not have any economic significance*, what matters is the *relative* magnitude of the *product* of PLC and ρ in comparison to WLC. For example, consider X = \$(100, 80) and Y = \$(100, 3). Without further information, no assessment can be made about the significance of markedly different PLCs (\$80 versus \$3) in X and Y. This is because ρ can be made arbitrarily large or small depending on the assigned Liability Index Thresholds. For example, if the LI Thresholds of owners of X and Y were 0.9 and 0.2 respectively, X would have more Buying Power than Y (\$91 versus \$88).

3.3 Imposition of Liability Index Range on Participating Countries

The third role of AAPL is to impose a Liability Index Range on every country that has adopted the non-scalar economy based on an agreed formula. The *Liability Index Range* is defined as a range of allowed LI Threshold values between a maximum and a minimum. The LI Thresholds of individuals, firms, and other entities within the country *must always be less than the upper bound* of the imposed LI Range and are *recommended* to be within this range. The actual LI Thresholds are chosen by the government based on their economic policy – see Figure 5.

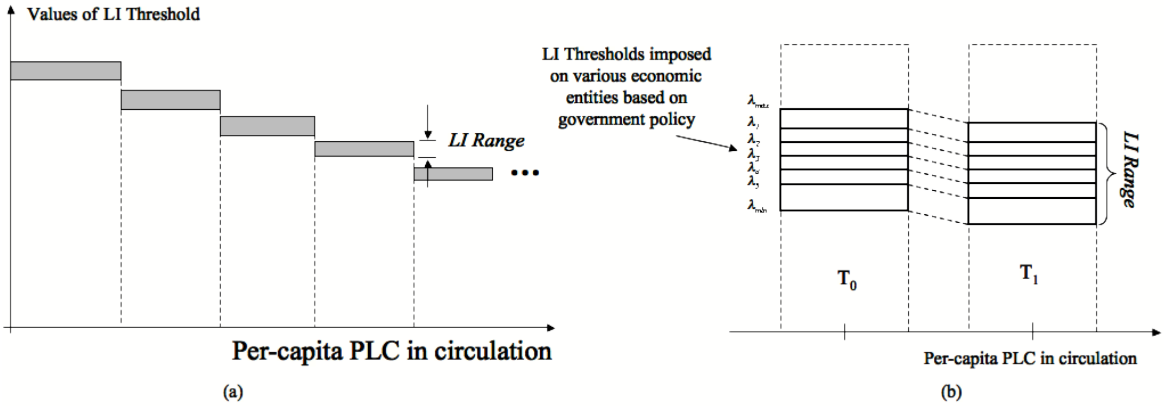

Figure 5: (a) Variation of LI Range in response to increases in per-capita PLC in circulation; (b) Mapping the LI Thresholds to new values in response to change of LI Range

For example, if the LI Range imposed on a country is [0.40, 0.45], the LI Thresholds of economic entities in this country cannot exceed 0.45. It is also recommended that these be in the inclusive range of 0.40 to 0.45. The government might assign everyone the same LI Threshold from this range or adopt a more sophisticated policy (Section 5).

AAPL determines the Liability Index Range of every country based on a formula agreed by all the participating countries. The exact nature of this formula is outside the scope of this paper. However, the LI Range is required to be a *monotonically decreasing function of per-capita PLC in circulation*. Figure 5-(a) shows one possible example where the LI Range is progressively decreased in response to the growth of per-capita PLC in circulation. Note that calculating the amount of PLC in circulation is straightforward for AAPL, because AAPL is the only authority responsible for creation and removal of PLC and can monitor the PLC transfer in/out of the economy due to import/export.

3.3.1 The impact of lowering the Liability Index Threshold of an entity

For an economic agent, lowering the LI Threshold has the effect of *increasing* the Marginal Cost of PLC - see Figure 3-(b). Consequently:

- For every dollar of PLC in money, more buying power is locked up. This applies *retrospectively* to all accumulated PLC.
- For every dollar of PLC in prices, the perceived Magnitude of price would be higher and more Buying Power is lost due to purchase.
- The perceived return is higher when purchasing an environmental good or investing in the environmental sector.

So altogether, a lower LI Threshold encourages reduced consumption, especially of goods and services that have high PLC in their prices, and more investment in environmental sector with the purpose of reducing one's accumulated PLC.

3.3.2 The impact of lowering the Liability Index Range of a country

Typically, when the LI Range of a country is changed, the LI Thresholds of economic entities are mapped to new values within the new range - Figure 5-(b). So, when the LI Range is lowered, everyone would experience a corresponding reduction of LI Threshold. Consequently, the whole economy must reduce consumption, especially of goods and services that are pollution intensive, and increase investment in the environmental sector.

The combination of these effects could trigger cooperative and competitive incentives to reduce the total PLC in circulation. In other words, the LI Range influences the balance between the amount of investment and effort devoted to pollution abatement and provisioning on one hand and production of goods and services for people on the other hand. *It determines the size of the environmental sector versus other sectors.*

4 Market Interactions

In non-scalar economy, two categories of firms can be distinguished. The first category or 'traditional' firms are called *wealth generating firms* and are engaged in production of goods and services for humans. A distinguishing feature of these firms is that the PLC of their prices and dividends is *non-negative*. The second category is referred to as *environmental firms*. These firms are engaged in environmental restoration and their products are referred to as *environmental goods*. The price of environmental goods and dividends of these firms have *negative* PLC.

Table 2 shows typical transactions for both categories. The first three transactions are similar:

1. The firm may *directly* be engaged in appropriation of global commons such as discharge of waste into the atmosphere or oceans. In this case, the pollution liability associated with these activities is created by AAPL and sold to the firm (Section 3.1).
2. The firm almost always creates pollution *indirectly* by purchasing other goods for its operation, such as electricity, transport, computers, raw materials, and so forth. The PLC associated with these is transferred to the firm via purchase operation.
3. The firm pays for its workforce (subtraction operation).

For wealth generating firms, the output is a product or service sold to the intended market. During the sell operation a certain amount of PLC as specified by the price is transferred to the firm's customers. Three cases can be identified:

a) If the PLC of price *under*-estimates the total pollution, directly and indirectly, caused by the firm's operation then the sell operation does not transfer the entire PLC to customers and there would be some accumulation of PLC within the firm. This PLC could either be transferred to the shareholders via dividends (who probably would not be pleased) or retained by the firm causing reduction of Buying Power and perhaps lowering the LI Threshold - Figure 8.

b) If the PLC of price *over*-estimates the pollution caused by the firm's operation, perhaps to compensate for the under-estimation of PLC during the previous offering, the firm depletes its accumulated PLC. However, customers perceive a higher Magnitude for the price and depending on their Marginal Cost of PLC may reject the product. Furthermore, the PLC of price clearly demonstrates to them that the higher Magnitude is due to polluting impact of firm's operation and not any other variation in cost factors.

c) If the PLC is accurately estimated, the pricing strategy is referred to as *neutral* pricing (neutral with respect to PLC accumulation). It can be shown that this is the best strategy in the long run. (The proof is outside the scope of this paper and will be subject of future publications.)

Table 2 also demonstrates that it is *easy* for the firm to adopt a neutral pricing strategy as the non-scalar economy provides all necessary information to work out the balance of PLC.

In the case of environmental firms, the provisioning activity results in transfer of PLC to AAPL. The firm may sell a number of environmental goods. The price of an environmental good has *negative* PLC. Purchasing an environmental good reduces the buyer's accumulated PLC and transfers certain amount of Wealth Component to the firm. The former helps the buyer to manage

Wealth Generating Firms	
Description	**Value**
Capital and assets	(x1, x2)
Pollution liability received directly from AAPL (purchase)	(0, a2)
Purchase of supplies (may include additional assignment of pollution liability at the point of sale or after purchase)	(b1, b2)
Payment of wages (subtraction)	(c1, c2)
Total revenue from sales (sell operation)	(d1, d2)
Balance	**(x1-b1-c1+d1, x2+a2+b2-c2-d2)**
Dividend returned to shareholder of the form (subtraction)	(y1, y2)

Environmental Firms	
Description	**Value**
Capital and assets	(x1, x2)
Pollution liability received directly from AAPL (purchase)	(0, a2)
Purchase of supplies (may include additional assignment of pollution liability at the point of sale or after purchase)	(b1, b2)
Payment of wages (subtraction)	(c1, c2)
Total pollution credit from AAPL Revenue for sale of environmental goods (sell operation)	(0, d2) (e1, -e2)
Balance	**(x1-b1-c1+e1, x2+a2+b2-c2-d2+e2)**
Dividend returned to shareholder of the form (subtraction)	(y1, -y2)

Table 2: The transactions of wealth generating and environmental provisioning firms

the PLC accumulation in his/her money. The latter compensates the firm for the cost of privately owned resources, such as labor and materials, used for its operation. For example, if the buyer's money is \$(100, 40) and his LI Threshold is 0.4 (Buying Power = \$40), purchasing an environmental good priced at \$(10, -10) results in \$(90, 30) with Buying Power of \$45.

The environmental firms do not generate wealth or Buying Power *directly* as these firms are focused on environmental restoration as opposed to providing amenities for humans. Their output unlocks *existing* Buying Power that has been made inaccessible due to PLC.

5 Equilibrium and Sustainable Points

In this Section, it is shown that a non-scalar economy with a monotonically decreasing function for variations of LI Range achieves an equilibrium state if at all feasible, that is, if the marginal cost is finite. The Equilibrium Point is defined as *a state of economy when the per-capita PLC in circulation is held at a constant level* (see Figure 4). At this point the marginal benefit of reducing per-capita PLC in circulation becomes equal to its marginal cost.

Let us denote the per-capita PLC in circulation by k. It can be shown that the marginal benefit of reducing k is bounded within the range of $[\rho, \hat{\rho}]$ where $\rho = (1-\lambda_k)/\lambda_k$, $\hat{\rho} = (1-\lambda_k^2)/\lambda_k^2$, and λ_k is

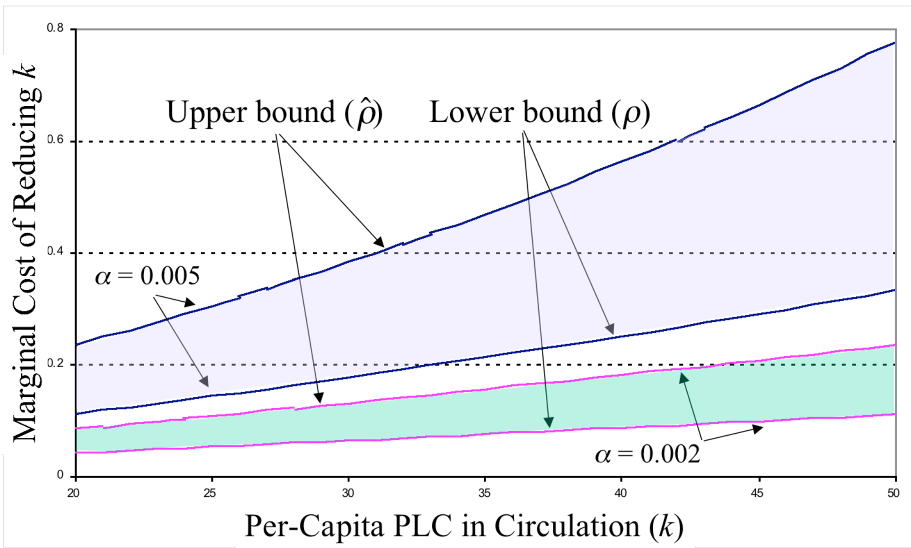

Figure 6: The bounds on the marginal cost of reducing per-capita PLC in circulation

the LI Threshold imposed at k. (The proof is outside the scope of this paper and will be subject of future publications.) As for the marginal cost of reducing k, the shape of this function cannot be ascertained at this stage and would depend on the efficiency of the environmental sector, level of competition, spending on R&D and advancements in technology. However, evaluation of this function would be quite feasible given the exact accounting information readily available when the non-scalar economy is operational. It is reasonable to assume that this is a monotonically *decreasing function* of k due to economies of scale and usually high establishment cost of infrastructure for abatement and provisioning.

Figure 7 shows the Equilibrium Points as the intersection points of a hypothetical marginal cost function with three possible marginal benefit functions based on different rates of lowering LI Threshold λ_k in response to rising k, denoted by α in the Figure. In this Figure, only the upper bound of marginal benefit ($\hat{\rho}$) is shown. The actual value would be somewhere between the upper and lower bound (ρ) as shown in Figure 6. Clearly, the marginal benefit of reducing k grows to arbitrarily large values in response to accumulation of PLC irrespective of unit prices associated with global resources. Hence, for any *finite* marginal cost and regardless of the shape of this function, there would be an intersection point.

This mathematical property of the marginal benefit function is desirable and matches the cumulative characteristics of pollution in the real world. The same polluting activity may have negligible impact in a clean environment (k small). However, if it occurs at the time when the capacity of the resource in absorbing further pollution is at its limit (k large), this extra pollution might be enough to 'push us over the edge' and trigger irreversible and undesirable changes. Similarly, in non-scalar economy, the economic impact of pollution is *amplified* in response to cumulative context of pollution. By tuning the parameter α, the economy models the sensitivity of the environmental resource to accumulation of pollution. For example, by lowering α, capacity of the resource is modeled to be higher and Equilibrium Point occurs at a higher value of k - Figure 7.

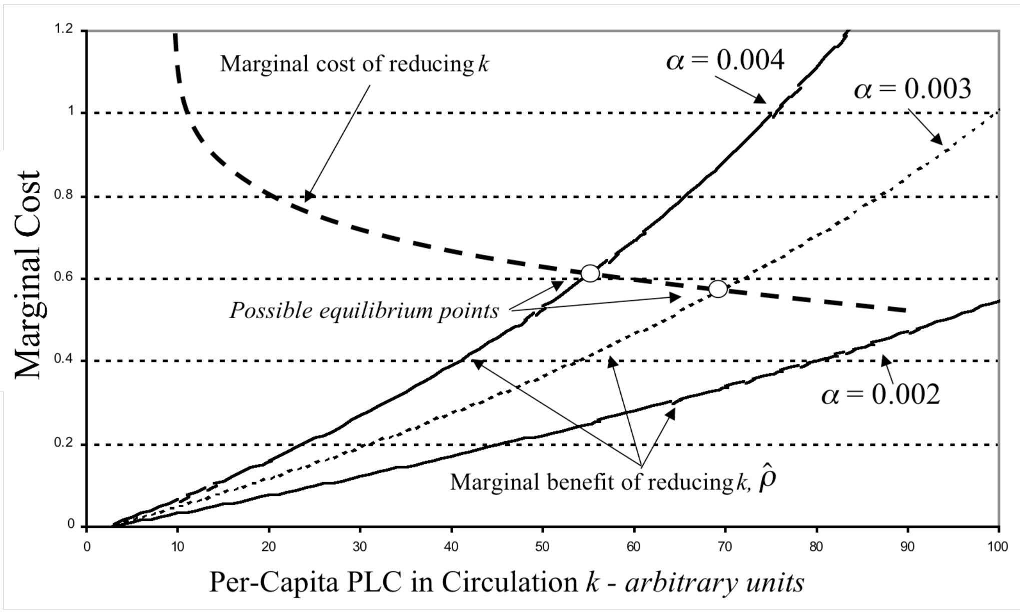

Figure 7: Equilibrium Points for different rates (α) of lowering LI Threshold in response to rising k

At the Equilibrium Point, the balance between provisioning, pollution abatement and appropriation activities are maintained at a level that does not lead to further accumulation of per-capita PLC. This does not imply that improvements in the standard of living are stopped. It merely states that the environmental sector has to *grow with the economy* or become more effective in reducing the polluting side effects of generating wealth.

The Equilibrium Point, however, may not be a *Sustainable Point* because the accumulated level of pollution at the equilibrium may still be excessive for the resource. It is however evident from the above discussions that the value of k at the Equilibrium Point can be altered by adjusting the parameter α above. In this way, by tuning a single parameter the society is able to steer the economy towards a sustainable point. This 'trial and error' method is also used in current schemes for establishing tax levels or environmental targets [1][3][8].

In the above derivation, we have assumed that everyone has the same LI Threshold within a given LI Range. While this is a possible policy for the government to adopt, a better incentive structure can be created by assigning different LI Thresholds to economic agents based on the level of pollution that they support. For example, the government policy may assign progressively lower LI Thresholds based on the accumulated PLC by the entity within the bounds of the LI Range - see Figure 8 for an example.

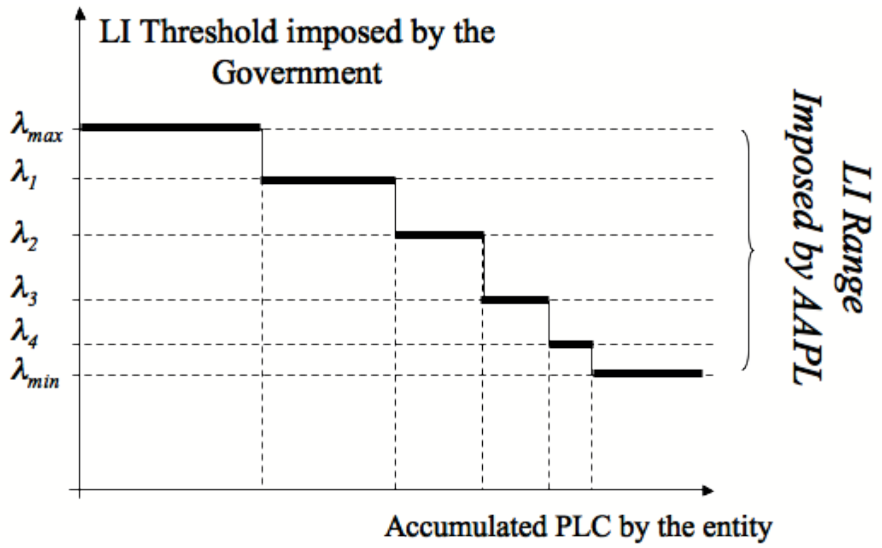

Figure 8: Possible variation of LI Threshold within the LI Range as a result of accumulation of PLC

6 Implementation Issues

Changing the underlying structure of price and money may appear disruptive and impractical. However, for a given environmental objective, implementation of non-scalar economy may be easier than current schemes for the following reasons:

1. **Electronic Transactions:** The non-scalar economy is most suited to electronic transactions, which are becoming almost ubiquitous. A change of software is required to implement the new mathematical model and its operations (addition, subtraction, purchase and sell). It is also necessary to attach every debit and credit account to a Liability Index Threshold monitored and adjusted by the regulator. However, these changes are quite feasible. It is also possible to support cash transactions in non-scalar economy (details are omitted for brevity).
2. **Monitoring infrastructure:** The existing infrastructure, deployed for other schemes could initially be used for non-scalar economy. There is no need to have everything ready on the first day and the scope of monitoring can be extended later. In general, the required monitoring infrastructure for non-scalar economy is similar to other schemes.
3. **No dilemma:** The introduction of non-scalar economy is not a zero-sum game and there is no need to have any losers because no particular sector or group is unfairly targeted.
4. **Backward Compatible:** (See below.)

5. **No international free riding:** (See below.)

In the remainder of this Section, more details on items 4 and 5 are presented.

6.1 Backward compatibility

All the mathematical operations of the non-scalar economy are reduced to the current system - and the intuitive understandings coincide - *when the Liability Index Threshold is set to one*. In this case, people can ignore the PLC of price and money as the Marginal Cost of PLC is zero.

Consequently, the non-scalar economy can be started by converting all monies and prices into two dimensional numbers with the second component set to zero. As there is no existing per-capita PLC in circulation ($k = 0$), AAPL would also set the Liability Index Range to [1.00, 1.00] which forces all Liability Index Thresholds to be 1.00. AAPL starts assigning pollution liability in major appropriation points that probably have existing infrastructure for monitoring. These include power generation plants, major mining and manufacturing firms and other pollution intensive industries. The unit price of global resources is selected to be a reasonable number but it does not need to be equal to the (unknown) cost of pollution.

The PLC generated by these firms appears in their prices and is transferred to buyers. At this stage, buyers show little resistance to PLC of price, as it has no economic impact. Nevertheless, the PLC gradually trickles down to others and circulates within the economy. After a while, everyone would have some PLC and the economy as a whole would have accumulated a certain level of per-capita PLC ($k > 0$). In response to this accumulation and based on a formula known to all, the Liability Index Range is lowered, say to [0.95, 0.97], which raises the Marginal Cost of PLC slightly and locks up some Buying Power. As this process continues, people's understanding of the new economy improves. They can also assess future opportunities in response to further lowering of Liability Index Range and adjust business practices, purchasing behavior and investment choices. The environmental sector experiences a significant growth and promotes innovation as investment in this sector becomes attractive. It is generally agreed that even current pollution regulations have resulted in the growth of pollution control sector [12]. After a period of transition, which can be made as long as necessary by altering the rate of variation of LI Range, the non-scalar economy diverges significantly from the current economy and a balanced strategy on appropriation and care for natural resources would have to be incorporated in every economic decision.

6.2 International free-riding

It is of course beneficial to start the non-scalar economy with an initial set of countries that are economically strong. These would agree on what resources to be managed and the parameters of model, and create AAPL for overseeing the appropriation and provisioning of these resources. Those countries that do not join, would incur some economic cost when trading with the non-scalar economy group. To illustrate this point consider the trade between two such countries, country A with non-scalar economy, and country B with current economy as shown in Figure 9. An exporter in B intends to sell some good to an importer in A. The scalar export price in B only reflects the cost of privately owned resources and is denoted by b_1. Of course a scalar price cannot be used in A, so AAPL has to translate this into a two dimensional price. Given that B's economy does not provide any information about the PLC of price, this component has to be estimated. One sensible approach would be to estimate the *average* Liability Index of the B's economy[8], denoted in the Figure by σ, and calculate the PLC of export prices of B using this common Index. In this case,

[8] Liability Index of an economy is defined as the Liability Index of the sum of all monies in the economy or alternatively as the ratio of the per-capita PLC in circulation over the sum of per-capita PLC and per-capita Wealth Component. Note that it is straightforward for AAPL to estimate this Index for a non-scalar economy. Likewise, based on industrial and environmental practices and observed per-capita PLC in circulation in non-scalar economies, it would be possible to extrapolate and provide a reasonable estimate for other (scalar) economies.

$b_2 = \sigma\, b_1 /(1-\sigma)$. This PLC is then *transferred to the importer in A* when the purchase takes place. Note the following:

- Transferring an estimated PLC to the importer in A is *not* a punitive measure or trade protection scheme to discourage imports from B. It is an impartial and honest attempt to provide a fair estimate for the PLC of price because the scalar economy does not provide the required information. After all, it would be neither fair nor correct to set this value to zero because almost no product can be produced without some level of PLC.
- AAPL is not interfering in the economy of B. The estimated PLC is transferred to an entity in A, who is bound by the rules of non-scalar economy.

In the other direction, consider an exporter in A with an export price of (a_1, a_2). As a result of this export, from the perspective of AAPL, a PLC to the value of a_2 would have to be transferred to the buyer in B. This is recorded by AAPL as a transfer to B's economy and is used to update the estimated Liability Index of B. The importer in B would probably not care about the PLC component of price and has no incentive to reduce the accumulation of PLC in B (in AAPL's books). Once again, recording the transfer of PLC during an export operation is not a punitive measure and is not interference in the affairs of B. It is simply an attempt to keep account of PLC, because a *sell operation can only cause transfer of PLC and is not a removal operation*.

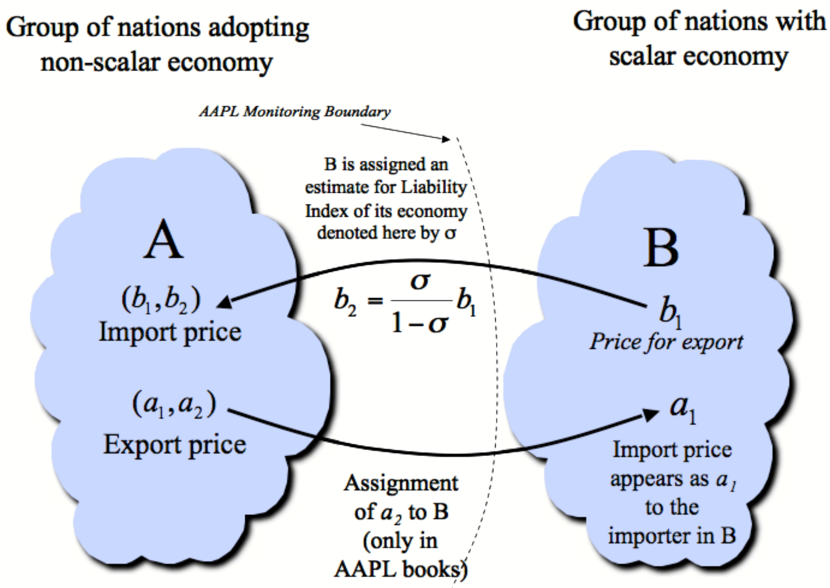

Figure 9: Translation between scalar and non-scalar prices during trade between scalar and non-scalar economies

As a result, the scalar economy B is disadvantaged. First, their export prices might look uncompetitive to buyers in A for reasons that are beyond the control of producers in B. Second, the economy of B is used as a 'dumping ground' of PLC by exporters in A which raises the average Liability Index of B even further, reducing the competitiveness of its exports. Because the economy of B is not equipped with proper mechanisms to control and manage the PLC accumulation, the situation would progressively get worse. The export industries in B starve and to produce cheaper goods may resort to environmentally inferior techniques. This would prompt AAPL to update its estimate of B's Liability Index and the cycle continues. The political and economic pressures may prompt some countries in the scalar group to revise their policy and adopt the non-scalar economy. By joining late, these countries have missed the early transition phase of implementation in synchronicity with other nations (Section 6.1). Their environmental sector is probably less developed and their transition is likely to be harder.

7 Comparison with other Incentive-Based Schemes

On the surface, it might appear that the distinctions between non-scalar economy and existing incentive-based schemes are merely notational. For example, the Magnitude of a price (a_1, a_2) is '$a_1 + \rho\, a_2$'. Is this really different from the scalar price a_1 that has been raised by '$\rho\, a_2$' due to pollution taxes or charges? When the Buying Power is equal to 'x_1 - $\rho\, x_2$', is this not the same as spending an extra '$\rho\, x_2$' dollars on charges associated with the environment? In this Section, we demonstrate that the differences between the two economies are profound with respect to both the accuracy of price signal and the economic incentives.

Let us consider the prices first. Assume that there are two products of same quality, priced at P = \$(80, 20) and Q = \$(20, 80). (We ignore other spurious influences such as brand loyalty.) To simplify comparison, let the buyer's LI Threshold be 0.5 (ρ = 1.0), which results in Magnitude of both P and Q to be \$100. Despite having the same Magnitude, buyers could easily recognise that P and Q production methods have affected the environment differently. In addition, *the economic impact of purchasing P or Q would also be different*.

It may be argued that similar information about the pollution effects of production could be provided in the current economy using mandatory labels accompanying the product. But determination of full effects of pollution in current economy is very difficult [7]. Production of goods causes pollution both directly and indirectly. While measuring the former is similar in both economies, the accounting for the indirect pollution would be error prone and quite complex in the current economy. The non-scalar economy accurately and automatically calculates the precise breakdown of price without any need for regulatory supervision. The incentives in the scalar economy, on the other hand, are probably in favour of *hiding* the true environmental cost of production, which is likely to lead to errors in estimation of this cost. These errors propagate when one product is used as an input for another production. The true contribution of environmental resources is diluted, obscured, and eventually lost among the 'noise' of other cost factors.

Therefore, in the non-scalar economy:

I. The price contains *precise* information about all the pollution caused by production, readily accessible to all and without a need for government mandates or supervision.

II. The accuracy of information in the price is *persistent* and is not lost in the course of economic transactions.

Consider a slightly different situation when the price for P is in fact \$(85, 20). With the scalar price, P is more expensive than Q (\$105 versus \$100) and would be rejected in the market. The producers of P could try to inform consumers about the merits of their environmentally friendly production technique. Consumers must first trust the propaganda and then face a dilemma, because a decision to forego their self-interest and buy the more expensive product would be *contrary* to the economic 'reality' signalled by the price. In the non-scalar economy there is no dilemma. Consumers can always be confident that an *economic* decision to purchase is consistent with their social obligations. They could safely ignore any advertising pleas for support and environmental-sounding names or logos. The price provides a full and accurate story.

Economic decisions, however, vary in the population and depending on the LI Threshold there could be some who are more sensitive to PLC in prices. If the LI Threshold of a market segment were lower than 0.5, say 0.47, the perceived Magnitudes of P and Q would be \$107.6 and \$110.2 respectively. This is not a subsidy for P. It is an economic differentiation influenced by the past buying behaviour and the level of pollution supported.

Therefore, in the non-scalar economy:

III. The information contained in the price alone is *sufficient* to provide a basis for an informed economic decision.

IV. Consumers can always use their economic judgment to support a product, as this is *consistent* with their social and environmental obligations.

V. It is possible to *differentiate* between people based on their history of supporting pollution.

Having established that the non-scalar economy is endowed with *superior signals*, we now turn our attention to *economic incentives*. In the current incentive-based schemes, the cost of natural resources is paid through taxes or purchase of permits and added to the cost of privately owned resources. By paying for the resource, the buyers could feel that their obligation toward the environment has been *transferred* to another party who has received the revenue and should now engage in undoing the damage. In non-scalar economy, on the other hand, the accountability for reducing PLC is *always held by the entity benefiting from the pollution*.

	Scalar Economy	**Non-scalar Economy (assume ρ = 1)**
Initial Money	\$500	\$(500, 0)
Price	\$400 (includes \$100 Pollution tax)	\$(300, 100)
Balance of Money after purchase	\$100	\$(200, 100)
Buying Power after purchase	\$100	\$100

Table 3: Comparison of two 'equivalent' purchases in scalar and non-scalar economies

Consider the two individuals of Table 3. For ease of comparison, the Liability Index Threshold of the individual in non-scalar economy is 0.5 (ρ = 1.0). Both start with an initial Buying Power of \$500 and purchase a good with the Magnitude of \$400, which includes \$100 due to 'cost' of pollution. After the purchase, both are left with a total Buying Power of \$100.

Even if we ignore all the previous discussions, economically, these individuals are *not* equivalent. They both posses the same Buying Power but they differ with respect to their total wealth, the signals received from the economy, and their incentives for future economic activity.

In the scalar economy, a \$100 'tax' has been paid and *lost*. The buyer would hope that the recipient of this revenue would use it to develop alternative production techniques or help the environment. *But no such guarantee is enforced by the economy*. In fact, revenues raised by pollution taxes are often diverted to other programs instead of pollution abatement [13].

The buyer in the non-scalar economy, on the other hand, has supported a product with \$100 of PLC. The transfer of this PLC to his money does not create revenue for anyone else. In this process, he does *not* lose any wealth either but \$100 of his Buying Power is locked up (based on the LI Threshold of 0.5). This can be unlocked provided he reduces this PLC. Reduction of PLC can *only* be achieved by working or investing on the environment, such as buying an environmental good, and cannot be substituted by any other activity. *This tight causal relationship is enforced by the economy as opposed to a social contract.* More importantly, the semantic separation between WLC and PLC and the fact that economic operations keep track of both components leads to *accumulation* of PLC in time. In this example, receiving \$100 of PLC might not be significant if viewed in isolation, but with the passage of time and going through many similar purchases, a large portion of the owner's Buying Power could be locked up.

Therefore, in the non-scalar economy:

VI. The accumulation of pollution liability is *visible* and has a *progressively higher* economic impact. Thus, of necessity, it leads to changes in behaviour and investment strategies.

VII. The economy is *accountable* for pollution created and *must* invest some effort in restoration and abatement.

VIII. Transfer of PLC does *not* generate revenue for anyone and, therefore, there are no perverse incentives in the economy.

IX. The responsibility to reduce PLC is shared by *all* and is not monopolised by governments or people of good will. In particular, the bulk of responsibility rests on people who benefit most from appropriation directly or indirectly.

We now summarise other comparative points that were alluded to in the previous sections:

X. In the non-scalar economy, the unit price of pollution to global resources can be set to an *arbitrary value* and there is no need to estimate the true cost of pollution (Section 3.2.1).

XI. Non-scalar economy provides a *unified framework* as opposed to a narrowly defined target for a particular pollutant. New pollutants and appropriation modes can be added to this framework and the *scope* of monitoring widened gradually. This may be viewed as a generalisation of 'bubble' and 'netting' provisions in current schemes [1].

XII. International implementation of non-scalar economy does not require a *strict* global consensus and does not suffer from the free rider problem (Section 6.2).

XIII. The non-scalar economy has been designed to be *backward compatible* with the current economy. So while the differences between the two economies might seem revolutionary, it is possible to *evolve* from the current economy gradually and without significant disruption (Section 6.1).

XIV. The proposed system is not 'hard coded' with a pre-conceived notion of sustainability and is not prescriptive about the target levels of environmental purity that is desirable. It provides appropriate macro-economic parameters for the society to *control and affect* the level of pollution at the Equilibrium Point (Section 5)

7.1 Co-existence with other schemes

There may be local environmental targets that are more suited to be managed using existing schemes. For example, the regulator may wish to impose a ceiling on emissions of a particular pollutant with detrimental effects on the local ecosystem using taxes, permits or command and control. This may be required because the non-scalar economy would achieve an overall reduction of pollution but may not be specific enough for this pollutant. In general, it is possible for other schemes to co-exist with the non-scalar economy.

8 Conclusions

This paper proposes an approach to remove the commons dilemma by redesigning the concept of money and price based on two-dimensional numbers. Despite the simplicity of the underlying mathematical model, the proposed scheme is effective in aligning the cooperative and competitive incentives to attain a sustainable balance between usage and care for the commons. In this scheme, there is a complete alignment between economic prosperity and sustainability. All else being equal, the economy mirrors the state of the environment. In other words, the economy is affected negatively when the environmental condition deteriorates and vice versa.

Moreover, the proposed model does not suffer from the *free rider* problem; does not require accurate estimation of the *true cost* of pollution; is *simple* to implement and *backward compatible* with the current economy.

References:

[1] Cropper, M. L., and Oates, W. E., "Environmental Economics: A Survey", published in the Economics of the Environment, 4th Edition, 2000, pages 55-106.

[2] Ostrom, E., Gardner, R. and Walker, J., "Rules, Games and Common-Pool Resources, The University of Michigan Press, 2003, page 56.

[3] Tietenberg, T. H., "Economic Instruments for Environmental Regulation", published in the Economics of the Environment, 4th Edition, 2000, pages 373-389.

[4] Porter, M. E., Linde, Claas van der, "Toward a New Conception of the Environment-Competitiveness Relationship", published in the Economics of the Environment, 4th Edition, 2000, page 180.

[5] Baden, J. A., and Noonan, D. S., Editors, "Managing the Commons" Indiana University Press, Second Edition 1998, page 66.

[6] ibid, pages 58, 101.

[7] ibid, page 68.

[8] Ostrom, E., "Governing the Commons: The Evolution of Institutions for Collective Action", Cambridge University Press 1990, pages 34, 38.

[9] Palmer, K., Oates, W. E., Portney, P. R., "Tightening Environmental Standards: The Benefit-Cost or the No-Cost Paradigm?", published in the Economics of the Environment, 4th Edition, 2000, pages 205, 214.

[10] Hahn, Robert W., "Economic Prescriptions for Environmental Problems: How the Patient Followed the Doctor's Orders", published in the Economics of the Environment, 4th Edition, 2000, page 438.

[11] Coase, Ronald, "The Problem of Social Cost", published in the Economics of the Environment, 4th Edition, 2000, pages 34-44.

[12] Jaffe, A. B., et al., "Environmental Regulation and the Competitiveness of U.S. Manufacturing: What Does the Evidence Tell Us?", published in the Economics of the Environment, 4th Edition, 2000, page 167.

[13] Stavins, Robert N., "What Can We Learn from the Grand Policy Experiment? Lessons from SO2 Allowance Trading", published in the Economics of the Environment, 4th Edition, 2000, pages 475, 478.